

Off-grid tessellations

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Abstract

Typical tessellations are repetitions of the same form over regular grids. We analyse notions of tessellations, ask “can it be folded, and what is the intuition behind?”, and consider how to make more “natural” figurative patterns using off-grid tessellations.

Grid tessellations

Origami tessellations are typically a repeated pattern over a regular grid of squares, triangles or hexagons. Possibly some other pattern is used near the borders (1.a), the pattern varies across the plane (1.b) or is non-flat (1.c). The patterns are geometrical and non-figurative.

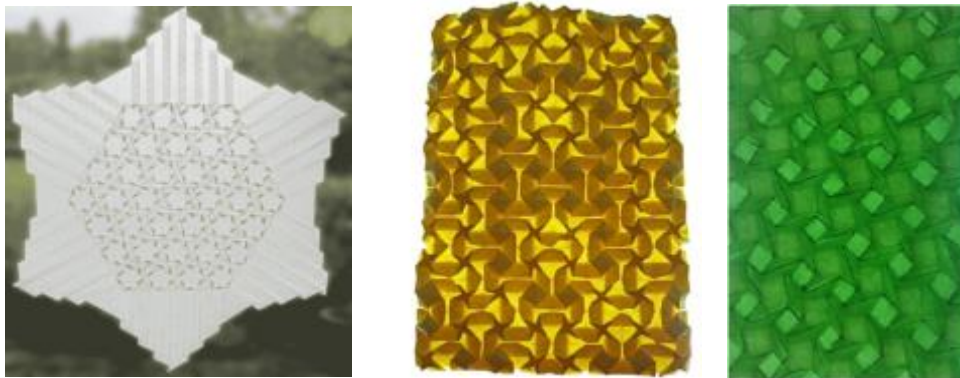


Figure 1: a) Triangular grid, hexagonal pleat molecules. b) Square grid, pleats, molecules vary. c) Square grid slanted, corrugated, negative and positive molecules. Design & fold: Hans Dybkjær.

Leaving the grid

A tessellation or *tiling* is a set of tiles placed in the plane such that there are no overlaps nor gaps. For a given tiling, the twist’n-fold algorithm by Batesman [Lang and Batesman 2011] makes it possible to fold a tessellation from that pattern. For flagstone tessellations [Cooper] they must obey the spider web condition [Lang 2015], essentially the tension equations of a weighted planar graph. The vectors at each node must sum to zero to be foldable, providing two equations with a free parameter for each edge, Figure 2a. Ternary nodes become rigid, higher arities have higher degrees of freedom.

Small changes may decide flagstone foldability, (2b-c), and symmetries play a role (2d). Flagstone foldability can be obtained in different ways such as making the tessellation more symmetric (traditional origami), or add extra edges to provide extra degrees of freedom.

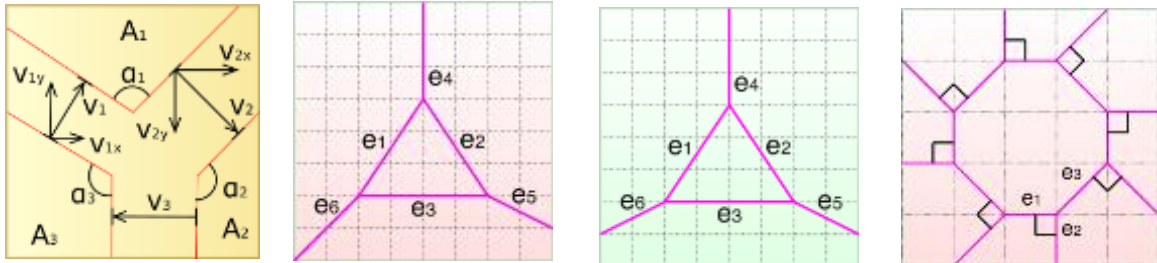


Figure 2: a) Flagstone folding tile movements must sum $v_1 + v_2 + v_3$ to 0. b) Not foldable. c) Foldable, symmetrical. d) Rotational symmetry fail, e1 must be both 1 and 16 wide.

Developing poinsettias

As an example of playing with the grids of tessellation designs, consider figure 3. Poinsettias are flowers of a 6-fold symmetry; actually, red leaves in a 3+3 symmetry. The first version (a) captures this as a pure star pattern over a triangular grid where a rhombic star pattern is completed with triangles to fill out the plane. The second version adds a centre to the flowers (b) and breaks the triangular grid, but still follows a regular pattern basically over that grid, using a set of three polygons with some cut-off along the raw paper edge. The third version (c) removes the grid and uses a set of free form polygons, no two alike.



Figure 3: Poinsettia designs. a) stars b) flowers c) figurative. Design & fold: Hans Dybkjær

The full paper

Will include a) math formulation and simple observations providing insight into off-grid foldability, b) node configuration and edge connectivity expanding Figure 2 including concave/convex issues and paper border issues, c) techniques for obtaining flagstone foldable patterns, d) CP's and better folded versions of the poinsettias, e) full references, and f) more.